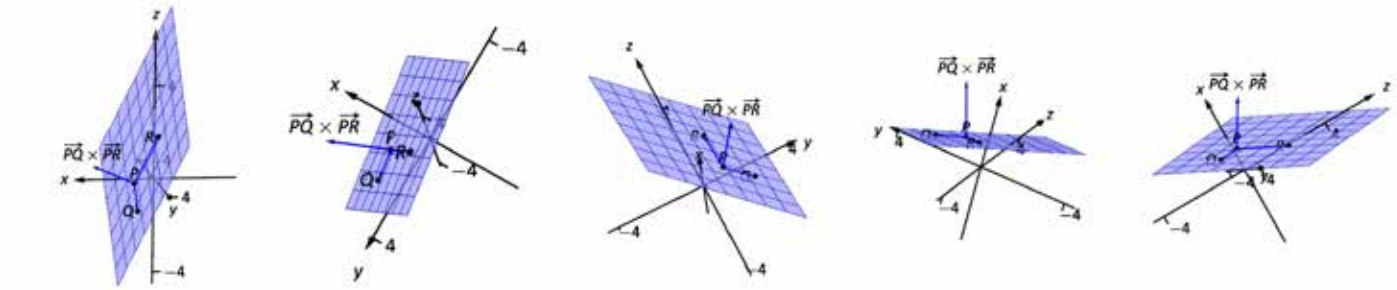

This image shows several perspectives of the graph of an equation for a plane, defined by three points in space, R,P, and Q, and the "normal vector," a ray that is always perpendicular to the plane, in blue, originating at P. These perspectives are just some of those available to cadets manipulating the graphic in Version 3 of the APEX calculus e-textbook. – *Graphic courtesy of Col. Greg Hartman.*

Graphics Go Interactive in VMI Calculus e-Textbook

By Sherri Tombarge

Calculus is the mathematics of change. It's also a hurdle nearly all math, engineering, and science majors at VMI must clear early in their cadetships. And the final leg of the calculus sequence asks cadets to create equations representing three-dimensional objects – a big leap from the two-dimensional mathematics they master in the first two semesters.

Assisting cadets in understanding these equations is the newest version of the VMI Department of Applied Mathematics' APEX calculus e-textbook, which offers interactive graphics. In earlier versions of the textbook, which was first released in 2012, drawings illustrating the equations were more or less the same whether cadets were using the print-on-demand version available for less than $15 or the PDF: flat drawings lying on a paper or virtual page.

"If I have a static picture of a three-dimensional object, which perspective do I show?" said Col. Greg Hartman, professor of applied math, who has spearheaded the e-textbook project. This was a key question as he developed earlier versions of the textbook. "When you take a three-dimensional object and put it in two dimensions, you lose something."

In Version 3, released last summer, cadets can select a drawing of a three-dimensional object and manipulate it, turning it from side to side or upside down. For shapes that are hollow in the center, the drawing can be turned so cadets can see inside the hollow. The equation – an abstract representation – becomes an object in space.

"It's hard to picture something you've never seen before," noted Hartman. With the interactive graphics, cadets get a more intuitive understanding of the shapes and relationships the equation represents. "You get a visual confirmation of what's going on in the whole scenario."

Hartman, who used the open-source graphics language Asymptote to create the graphics, pointed out that today's cadets, accustomed to accessing sophisticated graphical displays on their cell phones, expect to be able to manipulate graphics.

"It makes the math real," added Hartman. "When you're working on something, there's always this element of doubt. Sure, my answer was supposed to do something. Is it really right?"

Hartman and one of his APEX collaborators, Col. Troy Siemers, department head, had to do some learning of their own to create the graphics – Asymptote was brand-new to both of them.

"We were very excited when we first got the images to be interactive in a PDF – it took several failed attempts to get there," said Hartman.

Hartman was pleased to find Asymptote for the graphics because its open-source availability is consistent with the philosophy of the APEX textbooks. The source files for all of the textbooks are available online free of charge. Teachers and professors wishing to adapt the textbooks for their own classes are free to do so. The new graphics are equally adaptable.

Hartman notes that the interactive graphics use capabilities of the PDF file format not supported by all PDF readers. This means that anyone who wants access to the interactive capability must use the Adobe Reader, a program available as a free download.

The APEX calculus sequence is in use at VMI and at least 10 other schools. It has succeeded in its initial goal to take the money element out of textbook development. A Jackson-Hope New Directions in Teaching and Research Grant enabled Hartman to allocate his time to the project, and there are no substantial printing and distribution costs. Students get a textbook worth hundreds of dollars free or for a nominal cost.

And for Hartman, the work has been a pleasure, one that he'd like to pursue further. Future APEX projects may include an addendum to the calculus sequence that would include vector analysis, covered at VMI in Math 301, Higher Math for Engineers and Sciences.

"I'm all for writing that book," said Hartman. "It'd be fun for me for sure."

Information and downloads are available at *www.vmi.edu/apex*.

Global Math Competition

Taylor Thomas '18, Edward Olbrych '18, and Joseph Bruchalski '18, one of seven VMI teams, work on problems in the Mathematical and Interdisciplinary contests in Modeling hosted by the Consortium for Mathematics and Its Applications Jan. 29-Feb. 1. – *VMI Photo by Kelly Nye.*